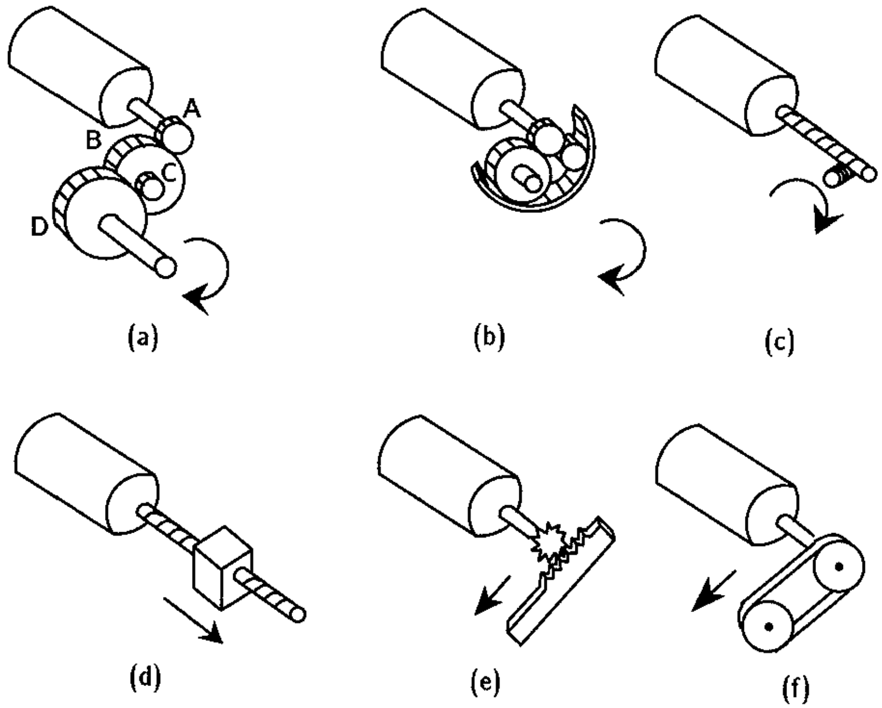
Power Transmission: Motivation



What kind of useful motion can we get at 2000 to 2500 RPM?

Linear Velocity and Force

Basic Equations:

$$v = 2\pi\omega r$$

$$F = \tau / r$$

Example Calculation:

$$\omega = 2500 \text{ RPM} \quad \tau_{max} = 3.26 \text{ oz-in} \quad r = 2 \text{ in}$$

$$v = 2\pi(2/12)2500 = 2619 \text{ ft/min}$$

$$v = 29.75\text{mph}$$

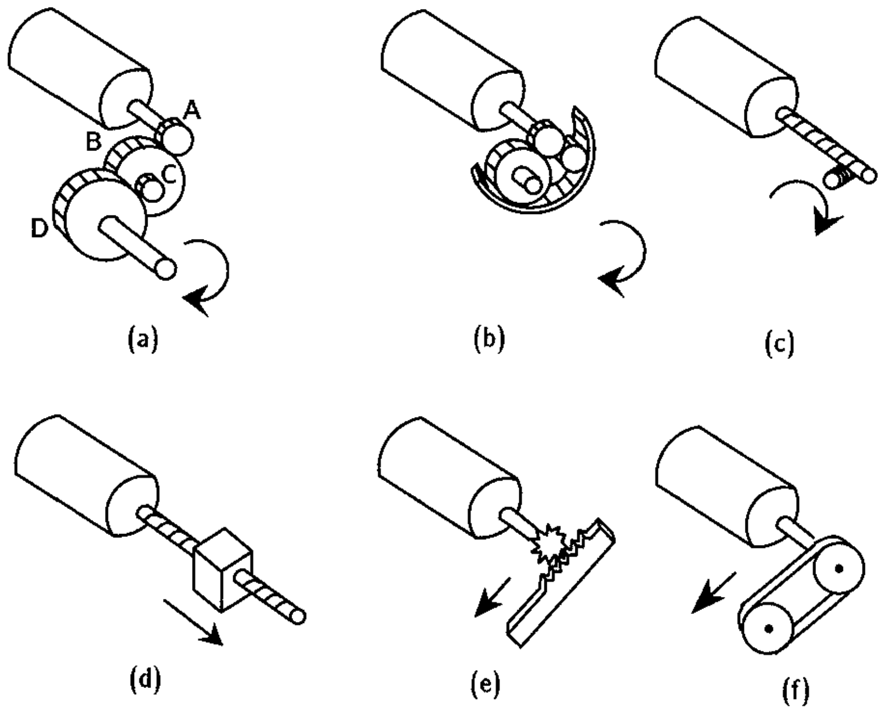
$$F = 1.63/2 = .815\text{oz (at max power)}$$

$$F = 3.26/2 = 1.63\text{oz (at stall)}$$

In order to get useful forces and motions at/near the maximum power/efficiency point of the motor, we need to make r smaller.

A transmission allows the designer to effectively reduce r while maintaining other design constraints.

Power Transmission: Basic Principle



Power In = Power out

$$\tau_{in}\omega_{in} = \tau_{out}\omega_{out}$$

$$F_{in}v_{in} = F_{out}v_{out}$$

Conservation of power(energy) allows us to make trade-offs between force and speed.

Transmission Ratio and Effective Radius

A transmission ratio is defined as:

$$r_t = \frac{\omega_{out}}{\omega_{in}} = \frac{\tau_{in}}{\tau_{out}}$$

For a compound transmission (gears, belts, chains) r_t has the form:

$$r_t = \pm \frac{\text{product of driver radii}}{\text{product of driven radii}}$$

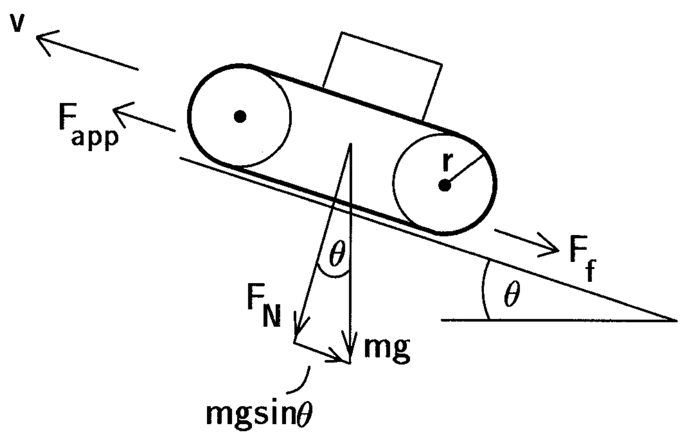
If we are interested in a particular output velocity or force, we have:

$$V_{out} = r_w \omega_{out} = r_w r_t \omega_{in} = r_e \omega_{in}$$

$$F_{out} = \frac{\tau_{out}}{r_w} = \frac{\tau_{in}}{r_t r_w} = \frac{\tau_{in}}{r_e}$$

r_e is called the effective radius.

Pushing



For a wheeled vehicle, the force available for pushing comes from friction.

Force balance:

$$F_{app} = \mu\, mg \cos(\theta) - mg \sin(\theta)$$

On a flat surface this is simply:

$$F_{app} = \mu mg$$

Since the motor is the driving force we can write:

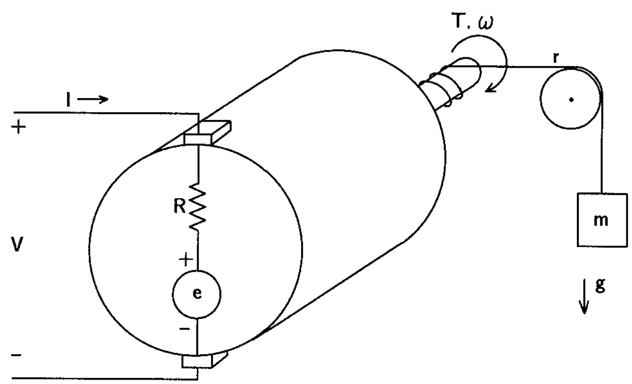
$$F_{app} = \mu mg \geq \frac{\tau_m}{r_t r_w}$$

This means

$$r_e = r_t r_w \geq \frac{\tau_m}{\mu mg}$$

otherwise slipping will occur (on a flat surface).

Lifting



How big do we make a wheel/pulley to lift the most weight at a fixed velocity?

Start with the motor equation:

$$\tau = \tau_{st} - \frac{\tau_{st}}{\omega_{nl}}\omega$$

Substitute in $\tau = W/r$ and $\omega := v/r$ and solve for W:

$$W = \frac{\tau_{st}}{r} - \frac{\tau_{st}}{\omega_{nl}}\frac{v}{r^2}$$

Maximize W by taking the partial w.r.t r and setting it to zero.

$$\frac{\partial W}{\partial r} = -\frac{\tau_{st}}{r^2} + \frac{2\tau_{st}v}{\omega_{nl}r^3} = 0$$

Solve for optimum r^* and W^*:

$$r^* = \frac{2v}{\omega_{nl}} \qquad W^* = \frac{\tau_{st}\omega_{nl}}{4v}$$

Transmission Design

1. Determine force and velocity requirements
2. Match motor to power requirements
3. Determine output wheel/pulley size limitations
4. Determine transmission ratio requirements
5. Select transmission elements based on power, ratio, and space requirements
6. Design transmission housing to support loads, minimize deflections, maintain alignments, etc.

Note: It is not unusual for items 1 and 2 to be reversed, that is, force and velocity requirements often come from pre-existing power specifications.